Tolling at a Frontier: A Game Theoretic Analysis

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Abstract:

Frontiers provide an opportunity for one jurisdiction to remedy inequities (and even exploit them) in highway finance by employing toll-booths, and thereby ensuring the highest possible share of revenue from non-residents. If one jurisdiction sets policy in a vacuum, it is clearly advantageous to impose as high a toll on non-residents as can be supported. However, the neighboring jurisdiction can set policy in response. This establishes the potential for a classical prisoner's dilemma consideration: in this case to tax (cooperate) or to toll (defect). Even if both jurisdictions would together raise as much revenue from taxes as from tolls (and perhaps more since taxes may have lower collection costs), the equilibrium solution in game theory, under a one-shot game, is for both parties to toll. However in the case of a repeated game, cooperation (taxes and possibly revenue sharing) which has lower collection costs is stable.

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1. INTRODUCTION

Tolls are viewed by transport economists as a more efficient means for financing highways and allocating scarce road space than general taxes in many cases (Bernstein and Muller 1983; de Palma and Lindsey 1998; Downs 1994; Dupuit 1849; Gittings 1987; Keeler and Small 1977; Mohring 1970; Poole 1994; Roth 1996; Small 1983; Small, Winston, and Evans 1989; TRB 1994; Verhoef, Nijkamp, and Rietveld 1996; Vickery 1963, 1969; Viton 1981, 1995). During some periods in the history of road financing, tolls have been widely used, including in the United States during the period from the late 1700s through the mid 1800s (Klein 1990), and again from 1940 - 1956 (Gomez-Ibañez and Meyer 1993). However most roads are now financed with gas taxes or from general revenue. If tolls ever again become a widely used revenue source, it won't happen overnight, they will be staged into wide acceptance. Some locations will be more politically acceptable for new toll collections than others. In particular, jurisdiction boundaries or frontiers, where at least half the crossing vehicles are driven by non-residents, would seem to be among the most politically palatable. However a frontier, by definition, involves more than one jurisdiction, and the policies of neighbors affect each other.[i]

This paper considers the welfare implications of tolling at a frontier under alternative behavioral assumptions: different objectives (welfare maximizing, profit maximizing, cost recovery), willingness to cooperate on setting tolls, and over different time frames (one-time interactions and repeated interactions). By understanding how tolls, welfare, and profits vary under different behavioral assumptions, we can better understand the motivations of jurisdictions and under which behaviors tolls will be most likely.

There are two problems that are considered in this paper, referred to as strategic and tactical decisions respectively. First is the strategic decision: will a jurisdiction tax or toll? Second is the tactical decision: if it tolls, what toll will it set? The decision to toll and the

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rate of toll set by one jurisdiction affects the welfare of the residents of another jurisdiction, leading to interactions and possible gains to both jurisdictions by cooperating. Game Theory, developed by Von Neumann and Morgenstern (1944), presents an analytic approach to explain the choices of multiple actors in conflict with each other with scope for cooperation, where the payoffs are interdependent (Axelrod 1984, Hargreaves-Heap and Varoufakis 1995, Osborne and Rubinstein 1994, Rapoport 1970, Taylor 1987).

The focus of this paper is on the revenue policies and rates of toll which emerge at jurisdiction boundaries under alternative behaviors in the absence of congestion. The model is developed in Section 2. Alternative objectives, one aspect of behavioral variation is considered in Section 3. Two different toll-setting methods, cooperative and non-cooperative are investigated in Section 4, and comprise the second main behavioral variation. Section 5 provides algebraic solution to the model under the different behaviors. Section 6 presents empirical values for the model, so that sensitivity tests may be conducted in Section 7 and the model applied in Section 8 in the context of a one-shot game. Section 9 extends the analysis of Section 8 into the realm of repeated games, where many outcomes are possible. Section 10 makes some concluding remarks.

2. MODEL

We assume an infinitely long two way road covered by two jurisdictions, one ranging from the point $-\infty$ to a boundary point b (jurisdiction J_I), the other covering the area from point b to $+\infty$ (jurisdiction J_J). Both jurisdictions may establish toll-booths at the boundary. Tolls can be collected in either one or both directions, which will affect welfare by a fixed amount associated establishing toll-booths and a variable cost per collection. For convenience we assume tolls in both directions if tolls are collected. There are no internal toll-booths. This is illustrated in Figure 1.

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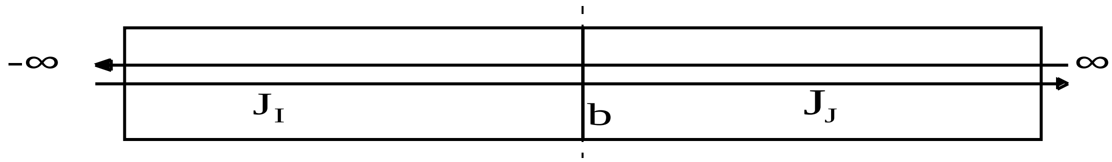
Figure 1 Infinitely Long Road Covered Completely by Two Jurisdictions



This network structure implies four classes of trips, trips staying within J_I (T_{ii}), trips from J_I to J_J (T_{ij}), trips from J_J to J_I (T_{ji}) and trips staying within J_J (T_{jj}). We are only concerned with trips crossing the boundary. By assuming symmetry, the equations for T_{ij} and T_{ji} trips are identical.

Our model assumes that flow (f_b) across point b on a road is described by a negative exponential model, where demand depends on the toll charged by both jurisdictions can be described by the function below.

$$f_b = \omega e^{\alpha(r_I + r_J)} \tag{1}$$

where: f_b = flow past point b

ω, α = model parameters

r_I, r_J = the toll charged by jurisdiction J_I , J_J

Because the jurisdictions are infinite in size, we are not interested in total welfare, rather only in welfare crossing the boundary point *b*. The consumer's surplus of local boundary crossing trips (U_{ij}) is measured as the difference between what each consumer would pay and what they do pay. We can solve for consumers' surplus by integrating the demand function over the range of tolls from what they do pay ($r_I + r_J$) to infinity. This is given by equation (2).[ii]

$$U_{ij} = \int_{r_I + r_J}^{\infty} \omega e^{\alpha(p)} \partial p = -\frac{\omega e^{\alpha(r_I + r_J)}}{\alpha} \tag{2}$$

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Two components comprise cost: network use cost (C_{Nij}) and toll collection cost (C_{Vij}). External costs are excluded because jurisdictions don't generally include them in their decision making. Implicit in this model is that jurisdictions have the obligation of maintaining a level of service with a specific travel speed. Thus "congestion effects" are ascribed to infrastructure costs which are proportional to traffic flow. To simplify the analysis we assume no (dis)economies of scale and we assume smoothly and continuously increasing infrastructure costs. We assume zero fixed costs associated with operating the network or collecting tolls or taxes. Equation 3 shows the network use cost (C_{Nij}), which equals the flow multiplied by the average trip length of the portion of the trip in jurisdiction I ($1/\psi$), multiplied by a cost per unit distance (φ).

$$C_{Nij} = \frac{\varphi}{\psi} \omega e^{\alpha(r_I + r_J)} \tag{3}$$

Equation 4 provides the cost of toll collection per traveler (C_{Vij}) as the flow multiplied by the collection cost per crossing (θ).

$$C_{Vij} = \theta \omega e^{\alpha(r_I + r_J)} \tag{4}$$

Equation 5 shows the revenue from toll collection (R_{ij}) as the rate of toll for jurisdiction I (r_I) multiplied by flow.

$$R_{ij} = r_I \omega e^{\alpha(r_I + r_J)} \tag{5}$$

3. OBJECTIVES

Which objective jurisdictions employ will shape the resulting tolls and welfare, and thus perhaps the decision to employ tolls. When it is assumed that jurisdictions have the objective of local welfare maximization, welfare is defined narrowly as the sum of profit (loss) from administering the road and consumers' surplus for its residents, as shown in equation (6).

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$$\underset{r_I}{Max} \quad W_L = U_{ij} + 2 * R_{ij} - 2 * C_{Nij} - 2 * C_{Vij} \tag{6}$$

The profit maximization objective excludes all consumers' surplus as given in equation (7) This represents conditions when the toll-booth is privately controlled, for instance to compare the consequences of unfettered private control with the public control of the network. To the extent that the welfare losses associated with private control are not excessive, it may be a reasonable organizational form for jurisdictions to consider.

$$\underset{r_I}{Max} \quad \Pi = 2 * R_{ij} - 2 * C_{Nij} - 2 * C_{Vij} \tag{7}$$

We can analyze the objective of local welfare maximization with a cost-recovery constraint. This objective requires that tolls be high enough to recover the costs imposed by those crossing the toll-booth but that toll revenue cannot be raised in excess of costs.

$$\begin{aligned} &\underset{r_I}{Max} \quad W_{LCR} = U_{ij} \\ &s.t. \quad 0 = 2 * R_{ij} - 2 * C_{Nij} - 2 * C_{Vij} \end{aligned} \tag{8}$$

Finally, we might for comparison purposes identify what would happen if both jurisdictions (J_I and J_J) were under single control. If that government imposes tolls, it will only require a single toll-booth, so collection costs will remain the same as a single jurisdiction. On the other hand, it will consider consumer's surplus of all frontier crossing trips and the network costs they impose on both jurisdictions roads.

$$\underset{r_I}{Max} \quad W_G = 2 * U_{ij} + 4 * R_{ij} - 4 * C_{Nij} - 2 * C_{Vij} \tag{9}$$

4. TOLL-SETTING

The discussion to date still leaves some latitude in how to solve the tactical problem of toll-setting. The issue, in solving for the toll of jurisdiction I (r_I) , is what toll (r_J) does jurisdiction I assume that jurisdiction J uses when it is known what policy they choose. Two approaches can be considered: non-cooperative and cooperative equilibria.

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First, if we assume no collusion (implicit or otherwise), we attain a non-cooperative Nash equilibrium for toll-setting. This means that Jurisdiction I can do no better by changing its toll given what Jurisdiction J does, while Jurisdiction J can also do no better. This does not necessarily result in the best satisfaction of the objective function, but is sustainable. This is solved keeping the two toll variables: r_I and r_J , separate and not necessarily equal.

It may be possible to attain higher overall welfare (profit) than non-cooperative approach. However it will be to the advantage of any jurisdiction to cheat (i.e. raise tolls) if the other jurisdiction doesn't cheat or retaliate but retains the cooperative tolls resulting from this solution. The cooperative solution is sustainable as an equilibrium in indefinitely repeated games.[iii] Simply, the issue again is how does Jurisdiction I treat r_J. To attain this cooperative solution, each jurisdiction includes both its own and the other jurisdiction's tolls as variables in its objective satisfaction calculations. (Under non-cooperative equilibrium, the other jurisdiction's toll could be treated as a constant). The overall payoff maximizing result can be achieved by setting $r_J = r_I$ in the equations, and solving for the equilibrium toll ($r^*=r_J=r_I$).

Economic theory argues that, when jurisdictions are welfare maximizing, cooperation should result in the rate of toll equal to the marginal cost of travel for those paying the toll, that is the network cost which is the average trip length of the portion of the trip in Jurisdiction I ($1/\psi$) multiplied by a cost per unit distance (φ) plus the cost of toll collection (θ). In fact, this is the case as will be seen in the next section.[iv] In the absence of fixed costs, and where average costs equal marginal costs, this implies cost recovery is satisfied.

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5. SOLUTIONS

Table 1 shows algebraic solutions for each scenario (combining objective and toll-setting methodology) assuming that jurisdictions do employ tolls. These results were simplified by assuming the demand coefficient α=-1. The final column shows the mathematical result assuming the empirical values described in the next section.

Table 1: Tolls by Scenario

Scenario (Objective: Maximize; Toll-Setting)	Solution	Result
Local Welfare, Non-Cooperative Toll-Setting (WN)	$r_I = \frac{2\varphi + \psi + 2\theta\psi}{2\psi}$	\$0.70
Local Welfare, Cooperative Toll-Setting (WC)	$r_I = \frac{\varphi + \theta\psi}{\psi}$	\$0.20
Local Profit, Non-Cooperative Toll-Setting (∏N)	$r_I = \frac{\varphi + \psi + \theta\psi}{\psi}$	\$1.20
Local Profit, Cooperative Toll-Setting (∏C)	$r_I = \frac{2\varphi + \psi + 2\theta\psi}{2\psi}$	\$0.70
Local Welfare, Cost Recovery Toll-Setting (CR)	$r_I = \frac{\varphi + \theta\psi}{\psi}$	\$0.20
Global Welfare Maximizing (WG)	$r_{IJ} = \frac{2\varphi + \theta\psi}{\psi}$	\$0.32*

note: solution obtained by setting α = -1, result obtained with empirical values described in Table 2.
** indicates tolls in case of Global Welfare Maximization, which should be halved to compare with other scenarios.*

The first thing to note is that the tolls resulting from the non-cooperative welfare maximizing scenario (r_I^{WN}) are the same as cooperative profit maximizing tolls ($r_I^{\Pi C}$). As mentioned in the previous section, we find that welfare maximizing cooperative tolls (r_I^{WC}) do equal the marginal costs of travel across the frontier. Also, because we have no fixed costs here, the tolls and welfare from the cost recovery objective is the same as welfare

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maximizing with cooperative toll-setting. The global welfare maximizing objective also has tolls equal to marginal costs, just that with fewer toll-booths, marginal costs are lower.

We realize some other interesting relationships in the analysis, independent of the empirical values of the model coefficients:

1. Profit maximizing cooperative tolls ($r_I^{\Pi C}$) are always \$0.50 higher than welfare maximizing cooperative tolls (r_I^{WC}).
2. Profit maximizing non-cooperative tolls ($r_I^{\Pi N}$) are always \$0.50 higher than welfare maximizing non-cooperative tolls (r_I^{WN}).
3. Welfare maximizing non-cooperative tolls (r_I^{WN}) are always \$0.50 higher than cooperative tolls (r_I^{WC}),
4. Profit maximizing non-cooperative tolls ($r_I^{\Pi N}$) are always \$0.50 higher than cooperative tolls ($r_I^{\Pi C}$).
5. Therefore, profit maximizing non-cooperative tolls ($r_I^{\Pi N}$) are always \$1.00 higher than welfare maximizing cooperative tolls (r_I^{WC}).

These relationships are summarized in Equation (10);

$$r_I^{WC} + \$1.00 = r_I^{WN} + \$0.50 = r_I^{\Pi C} + \$0.50 = r_I^{\Pi N} \tag{10}$$

In contrast with the usual application of cooperative equilibria for analyzing industrial organization of competitive markets, the best repeated game (cooperative) equilibrium toll is lower than the Nash equilibrium (non-cooperative) toll. Furthermore, the lower toll results in higher welfare and profit. The main reason for this is that we are dealing with complementary rather than substitute goods in our revenue mechanism game. Thus, cooperation to lower tolls allows higher welfare in an application similar to serial monopolists raising profits by cooperating to lower tolls (Chamberlin 1933). A second reason is that the objective function includes not just profit, but also consumers' surplus.

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6. EMPIRICAL VALUES

The model does not have much real-world meaning without understanding typical values for the model coefficients. Table 2 gives some values developed from earlier research by the author (Levinson 1998). The first two variables, α and ω describe demand. The variable α is set to -1, this value makes consistent what is known about the user costs of highway travel developed from a full cost study (Levinson and Gillen 1998) and a gravity model's decay function (Levinson and Kumar 1995). This variable must be less than zero to ensure that demand falls when prices rise. The second demand variable ω describes the number of trips when the total monetary price $r_I + r_J = 0$. Clearly this is a scalar and does not affect tolls or the ultimate decision to tax or toll in this analysis. To keep this analysis consistent with other research by the author, it is set at 2338, which is a value derived from a more complex version of the model (considering multiple jurisdictions). The variable network cost is the cost that a jurisdiction faces for every vehicle kilometer traveled. The value of φ=0.018 was estimated by Levinson and Gillen (1998) from a database of state highway expenditures and vehicle travel. The variable collection cost (θ) was estimated from toll collection costs on California bridges (Levinson 1998). Average trip length ($1/\psi$) within the jurisdiction was calculated from the multiple jurisdiction model, which required a factor for which trips were sensitive to distance traveled, (ψ=\$0.15/km), developed in Levinson and Gillen (1998).

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Table 2 : Empirical Values of Model Coefficients

Variable	Description	Value
alpha (α)	coefficient relating demand to price	−1
omega (ω)	demand multiplier (trips at price =0)	2338
phi (φ)	variable network cost ($/vkt)	0.018
1/psi ($1/\psi$)	average trip length in jurisdiction (km)	6.67
theta (θ)	variable collection cost ($/vehicle)	0.08

7. SENSITIVITY TESTS

Figures 2 through 5 show sensitivity of the model as we vary key parameters around their assumed variable (shown in Table 2). Table 3 gives us the elasticity (the percentage change of the variable of interest: tolls, profits, and welfare for each percentage change in the input variable for each scenario. Figures 2, 3, and 4 illustrate how tolls rise linearly as unit costs (θ, φ) and trip lengths ($1/\psi$) rise, keeping all other variables at the values shown in Table 2. Figure 5 shows how welfare and profits change as tolls ($r_I=r_J$) vary, again assuming all other variables are at the values shown in Table 2. Welfare is maximized when tolls are $0.20, profits when tolls are $0.70. Clearly when collection and network unit costs rise, welfare and profits decline. Trip lengths are somewhat more complicated, as they rise, tolls rise but so does welfare and profit until trip lengths exceed 66 km.

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Table 3 : Elasticity of Tolls, Profits, and Welfare as Inputs Vary

	Toll: W-NONC	Toll: ∏-Nonc	Toll: W-Coop	Payoff
Trip Length	0.171	0.100	0.600	1.765
Network Costs	0.171	0.100	0.600	-0.215
Collection Costs	0.114	0.067	0.400	-0.159

note: the elasticity of payoffs to changes in trip length, network costs, and collection costs is the same for both welfare and profit, and cooperative and non-cooperative equilibria

8. NON-COOPERATIVE GAME THEORY

Non-cooperative game theory is employed to analyze the strategic interactions between two jurisdictions under various conditions and objectives. Two decisions are considered: first, the strategic choice of revenue mechanism (tax or toll); and second, the tactical selection of the rate of tax or toll given the strategic choices by jurisdiction J_0 and the other jurisdictions (the environment).

The application of game theory requires acceptance of certain assumptions about the behavior of actors (in this case jurisdictions) and their level of knowledge. First, it is assumed that actors are instrumentally rational, that is they express preferences (which are ordered consistently and obey the property of transitivity) and act to best satisfy those preferences. Second, it is assumed that there is common knowledge of rationality (CKR), which means that each actor knows that each other actor is instrumentally rational, and that each actor knows that each actor knows, and so on. Third, it is assumed that there is a consistent alignment of beliefs (CAB), such that that each actor, given the same information and circumstances, will make the same decision - no actor should be surprised by what another actor does. Last, it is assumed all players know the rules of the game, including all possible actions and the payoffs of each for every player. These four assumptions are used in our analysis of a highly stylized game between two jurisdictions who have clear objectives.

The payoff to each jurisdiction depends on the policy (tax or toll), objective (welfare or profit), and the toll-setting equilibrium (cooperative or non-cooperative) taken

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by both itself and the other jurisdiction. The source of interaction between jurisdictions derives from residents of one jurisdiction traveling on the roads of the other. Thus the revenue and the pricing policy of one jurisdiction alters the demand for the roads of both jurisdictions. The payoffs to jurisdictions are shown in Tables 4 and 5, representing Welfare and Profit respectively.

Table 4 : Payoffs for Welfare Maximizing Jurisdictions

J_I \ J_J	∏-Non-Coop.	W-Non-Coop. = ∏- Coop.	W-Coop. = Cost Recovery	Tax
∏-Non-Coop.	[636, 636]	[1049, 699]	*[1822, 577]*	*[2226, 535]*
W-Non-Coop. = ∏- Coop.	[699, 1049]	[1153, 1153]*	*[1901, 951]*	*[2322, 883]*
W-Coop. = Cost Recovery	*[577, 1822]*	*[951, 1901]*	*[1567, 1567]*	*[1914, 1455]*
Tax	*[535, 2226]*	*[883, 2322]*	*[1455, 1914]*	*[1777, 1777]*

note: [payoff to J_I, payoff to J_J]; *: Indicates Nash Equilibrium in One-Shot Game; *Italics*: Indicates Higher Welfare Scenario Pair; *Underline Italics* : Indicates Highest Welfare Scenario Pair with Toll Policy, Stable under repeated game equilibrium; *Double-Underline Italics* : Indicates Highest Welfare Scenario Pair

Examining Table 4, we can find the Nash equilibrium solution to the one-shot game, that is the solution where J_I cannot improve its position given what J_J is doing, and vice versa, for welfare maximizing jurisdictions. The tolls from the non-cooperative local welfare maximizing scenario produce the Nash Equilibrium. For all J_J polices, J_I maximizes welfare by choosing this policy, similarly for J_J. However, a number of scenario pairs, denoted in italics have higher overall welfare, both jurisdictions together would be better off if somehow they could choose any of those pairs. Assuming toll policies, welfare would be maximized by each jurisdiction choosing the lower tolls of cooperative toll-setting, while overall, a [tax, tax] scenario pair (with no tolls) has the highest overall welfare.

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Table 5 : Payoffs for Profit Maximizing Jurisdictions

J_I \ J_J	∏-Non-Coop.	W-Non-Coop. = ∏- Coop.	W-Coop.= Cost Recovery	Tax
∏-Non-Coop.	[424, 424]*	*[699, 350]*	*[1246, 0]*	*[1521, -169]*
W-Non-Coop. = ∏- Coop.	*[350, 699]*	*[577, 577]*	*[951, 0]*	*[1161, -279]*
W-Coop. = Cost Recovery	*[0, 1246]*	*[0, 951]*	[0, 0]	[0, -459]
Tax	*[-169, 1521]*	*[-279, 1161]*	[-459, 0]	[-561, -561]

note: *: Indicates Nash Equilibrium in One-Shot Game; *Italics* : Indicates Higher Profit Scenario Pair; *Underline Italics* : Indicates Highest Stable (non-cooperative repeated game) Profit Scenario Pair; *Double-Underline Italics* : Indicates Highest Profit Scenario Pair

Similarly, examining Table 5, where both jurisdictions are profit maximizing, we find that the Nash equilibrium is to employ the tolls assuming profit-maximizing non-cooperative toll-setting. Again, a number of scenario pairs have higher overall payoffs.

Table 6: Payoff Accruing to Jurisdictions: J_I Welfare Maximizing, J_J Profit Maximizing

J_I \ J_J	∏-Non-Coop.	W-Non-Coop. = ∏- Coop.	W-Coop. = Cost Recovery	Tax
∏-Non-Coop.	[636, 424]	*[1049, 350]*	*[1822, 0]*	*[2226, -169]*
W-Non-Coop. = ∏- Coop.	[699, 699]*	*[1153, 577]*	*[1901, 0]*	*[2322, -279]*
W-Coop. = Cost Recovery	*[577, 1246]*	*[951, 951]*	*[1567, 0]*	*[1914, -459]*
Tax	*[535, 1521]*	*[883, 1161]*	*[1455, 0]*	[1777, -561]

note: *: Indicates Nash Equilibrium in One-Shot Game; *Italics* : Indicates Higher Payoff Scenario Pair; *Double-Underline Italics* : Indicates Highest Payoff Scenario Pair

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Combining the matrices from Table 4 and Table 5, shown in Table 6, we consider the payoffs where one jurisdiction is welfare maximizing J_I and the other J_J is profit maximizing. In this case, for a one-shot non-cooperative equilibrium game, J_I chooses the welfare maximizing non-cooperative tolls while J_J chooses the profit maximizing non-cooperative tolls. Most of the other scenario pairs produce higher total payoffs, indicating gains from cooperation or a repeated game.

9. INFINITELY REPEATED GAME

Tables 4, 5, and 6 represent a number of payoffs, but at their heart lie a complex prisoner's dilemma, with multiple cooperative and non-cooperative strategies. The tables show that the Nash equilibrium solution does not have the highest overall payoff. In a repeated game, the payoff maximizing solution may also be an equilibrium when some mechanism to enforce cooperation is in place. Cooperation has two advantages. First cooperation protects local citizens from the negative effects of other jurisdiction's pricing policies. Second, cooperation eliminates the finance externality which reduces demand for local roads from non-local residents and then hurts profits. Other mixed policies (alternating [Tax, Toll] and [Toll, Tax] for instance) may also achieve higher results, especially since they reduce collection costs and the negative effects of a serial monopoly relative to a single monopoly (Chamberlin 1933). Enforcement mechanisms include the ability to "punish" and "reward" neighbors in a repeated game, a government in the case of many players (jurisdictions), or a negotiated treaty, contract, or compact.

This dissonance between individual and collective payoffs in a one-time game may disappear in a repeated game. While both the one-shot and the finitely repeated prisoner's dilemma give unique solutions, the indefinitely repeated prisoner's dilemma does not ensure a unique solution. The "Folk Theorem" demonstrates that in infinitely and indefinitely repeated games, any of the potential payoff pairs in repeated games can be obtained as a Nash equilibrium with a suitable choice of strategies by the players. There

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are always multiple equilibria in an indefinitely repeated game, though some strategies have higher collective payoffs than others. Given various discount rates, different solutions will result in the highest repeated game payoff.

The question is how cooperation between jurisdictions can be achieved. A mechanism which can result in strategic cooperation without actual negotiation is the enforcement available in repeated games. In an indefinitely repeated game, one jurisdiction's behavior can be disciplined by another. Cheating on an agreement (for instance tolling when taxing was agreed to) by jurisdiction J_I in one round (year) can be punished in the next period by jurisdiction J_J, which would also toll, thereby hurting the payoff to jurisdiction J_I. This section applies the mathematics underlying repeated games, and computes the necessary discount factors for cooperation to be stable between rational jurisdictions.

To begin we will examine the conventional two strategy one-shot game. Consider the representation in Table 7 (after Taylor 1987) of the payoffs for two strategies of the two player prisoner's dilemma game, where the traditional prisoner's dilemma cooperate strategy is associated with tax and the defect strategy with non-cooperative toll-setting. (A similar construction could be made between either of these two policies and a cooperative toll-setting policy). As noted above non-cooperative toll-setting is a Nash equilibrium in this one-shot game. The letters w, x, y, and z are used to denote the payoffs in this section as shown in the table.

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Table 7: Welfare of Boundary Crossing Trips on Infinite Road Covered by Two Welfare-Maximizing Jurisdictions

$J_0 \setminus J_1$	Tax	Non-Cooperative Tolls
Tax	[x, x] = [1777, 1777]	[z, y] = [883, 2322]
Non-Cooperative Tolls	[y, z] = [2322, 883]	[w, w] = [1153, 1153]

where: y > x > w > z, numeric values indicate payoff from model

Payoffs from repeated games (or a supergame) can be thought of as the summation of a series of payoffs from one-shot games, discounted so that the present period's game is more valuable than the next and so on. If we define a discount factor for jurisdiction i, a_i, (and a discount rate: $1- a_i$), then we can compute the supergame payoff (X) from a strategy which results in the payoff x on every turn as $X = x(a_i + a_i^2 + a_i^3 + \ldots)$, or $X = x(a_i / (1- a_i))$, and similarly for any other payoffs (w, y, z). It should be noted that $1 \geq a_i \geq 0$, and other values are invalid (suggesting either future payoffs are more valuable than the present if $a_i > 1$, or that future payoffs are negative in value if $0 > a_i$). It should also be noted that the discount factor can vary for different jurisdictions.

Strategies in a sequence of games can be formulated which result in stable equilibria for each player and higher payoffs. We will consider four supergame strategies: tax on every round (χ^∞), toll on every round (τ^∞), conditionally tax with initial trust (B), and conditionally tax with initial distrust (B'). The first conditional strategy (B), (also called *tit-for-tat*) begins by cooperating (imposing a tax) on turn 1, and then on all subsequent turns does what the other player did in the previous turn. A variation on this strategy (B') is also *tit-for-tat*, but begins by defecting (imposing a toll) on turn 1, and then doing what the other player did.

We can conclude that in the repeated game, the strategy pair of both jurisdictions choosing to toll on every round, independent of what the other players are doing, $[\tau^\infty, \tau^\infty]$, is an equilibrium. Neither player can improve their position if the other plays τ^∞.

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However, this is not necessarily the best equilibrium. The strategy of taxing every round, again independent of what the other players are doing (χ^{∞}), is never an equilibrium. If your opponent is playing χ^{∞}, there is always a gain possible from any other strategy. The conditional supergame strategies, where the policy employed by one jurisdiction depends on what other jurisdictions did on a previous turn, are more complicated.

We can reformulate the game in terms of supergame strategies, shown in Table 8. The three supergame strategies which are sometimes equilibria (B, B', τ^{∞}) can be played by jurisdiction J_I and J_J. The cells in the table show which conditions (of Table 9) hold for the supergame strategy to be a repeated game equilibria. It can be shown (Taylor 1987) that the results shown in the first column of Table 9 hold when the conditions in the second column bear out.

Table 8: Conditions for Supergame Strategies to be Equilibria

J_0 \ J_1	B	B'	τ^{∞}
B	(1) & (2) for J_0, J_1 [$1 \geq a_i \geq 0.60$]	(3) & rev. (2) for J_0, J_1 [$0.60 \geq a_i \geq 0.23$]	Never equilibrium
B'	(3) & rev. (2) for J_0, J_1 [$0.60 \geq a_i \geq 0.23$]	(4) & rev. (3) for J_0, J_1 [$0.30 \geq a_i \geq 0$]	(4) & rev (3) for J_J [$0.30 \geq a_J \geq 0$]
τ^{∞}	Never equilibrium	(4) & rev (3) for J_I [$0.30 \geq a_I \geq 0$]	Always equilibrium

Note: rev. denotes reversing the ≥ in the equation (i.e. making it ≤). Conditions are defined in Table 6.8

[] indicates results of conditions for game

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Table 9 : Conditions for Supergame Strategies, and Results from Equations Above

Result	Condition	Value of RHS
(1) B is superior to τ^{∞} if	$a_i \geq \frac{y-x}{y-w}$	0.46
(2) B is superior to B' if	$a_i \geq \frac{y-x}{x-z}$	0.60
(3) B' is superior to τ^{∞} if	$a_i \geq \frac{w-z}{y-w}$	0.23
(4) Mutual B' is stable if the reverse of condition (3) holds and	$a_i \leq \frac{w-z}{x-z}$	0.30

The final column of Table 9 gives the value associated with the right hand side of the condition in the table. Applying those conditions to the strategy pairs of Table 8 we get the solution to the repeated game equilibria, shown by the range of discount factors shown in brackets in that table. We assume that if there are multiple equilibria in the game, that jurisdictions will choose the one which results in the highest welfare to them so long as it results in the highest welfare to other players. Just as in the one-shot game, if there is one stable equilibrium which does provide the highest welfare to all players, it can be anticipated to be chosen. We see several policy pairs are valid (repeated game equilibria). Significantly for discount factors in the range: $1 \geq a_i \geq 0.60$ (or discount rates between 0% and 40%, where typical governement interest rates are well under 10% in the United States in the 1990s), mutual cooperation [B, B] is a stable equilibrium, and since it has the highest payoff, we can assume that it would be the selected equilibrium.

This alternating policy pair [B', B] or [B, B'] emerges as stable for the range of discount factors: $0.60 \geq a_i \geq 0.23$ (or discount rates between 40% and 77%). Implicitly this assumes that toll-booths can be constructed and removed at no loss, or at least result in no charge during the off-turn, though the extent to which this is true is empirical. A similar policy is for one jurisdiction to always play cooperate and the other defect, so long as

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revenues are shared equally between them. Whether this can actually be enforced depends on the institutional arrangements between the jurisdictions. However, if we assume that these jurisdictions can cooperate at that level, it is unclear why they would select the alternating policy pair unless it had a higher payoff.

A range of discount factors ($0.30 \geq a_i \geq 0$) (discount rates between 70% and 100%) allows the policy pair of [B', B'] to be stable, which in practice is the equivalent of mutual defection [τ^{∞},τ^{∞}]. Similarly [τ^{∞}, B'] and [B' ,τ^{∞}] are stable when one or the other jurisdiction has such a low discount factor ($0.30 \geq a_i \geq 0$). These policies are also the equivalent of mutual defection [τ^{∞}, τ^{∞}].

This exercise can be undertaken for other profit and welfare maximizing policy couplets. The key point to take away is that cooperative equilibria are stable for a wide variety of realistic interest rates for indefinitely and infinitely repeated games.

10. SUMMARY AND CONCLUSIONS

This paper examined the question of what happens when jurisdictions have the opportunity establish toll-booths at the frontier separating them. Clearly, tolls are more likely at frontiers than at internal locations if only because a greater percentage of the toll falls on non-residents. Nevertheless, for larger jurisdictions, frontier toll-booths still raise nearly half their revenue from residents.

If welfare-maximizing jurisdictions behave non-cooperatively, they are likely to toll, however if they can arrange to cooperate, they will employ lower tolls or agree not to toll. Cooperation is easier the fewer jurisdictions involved. A border between two large jurisdictions essentially involves traffic from only those two jurisdictions. However, that same border along small jurisdictions will serve traffic from many different jurisdictions.

If all jurisdictions hope to maximize profit, they will toll, even if they do cooperate. However if they cooperate, they will charge lower tolls and even eliminate one toll-booth

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between them (so that they share revenue while lowering operating costs). Profit maximization is more likely under private sector management than public sector. So if tolling is a desired policy outcome, privatization will be more likely to achieve it than public control.

There are several ways the analysis could be extended. First is the inclusion of congestion costs. Congestion pricing is often cited as the main benefit from road pricing, but its benefits cannot be understood with the model in the absence of delay due to excess demand. Second, this paper has assumed that travelers are identical except in their reservation price. Congestion pricing is most meaningful when demand is heterogeneous, that is different travelers have different values of time and differ in their disutility from congestion. Third, all fixed costs were neglected. This simplifies the analysis, particularly under cost recovery behavior, but is not necessarily a realistic approach.

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Figure 2 : Tolls as Network Cost Changes by Scenario

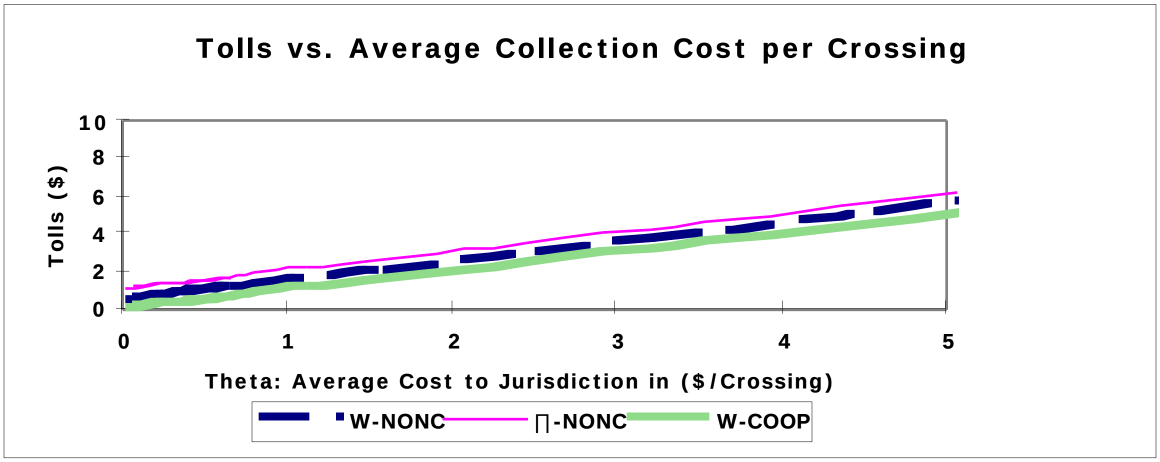

Figure 3 : Tolls as Collection Cost Changes by Scenario

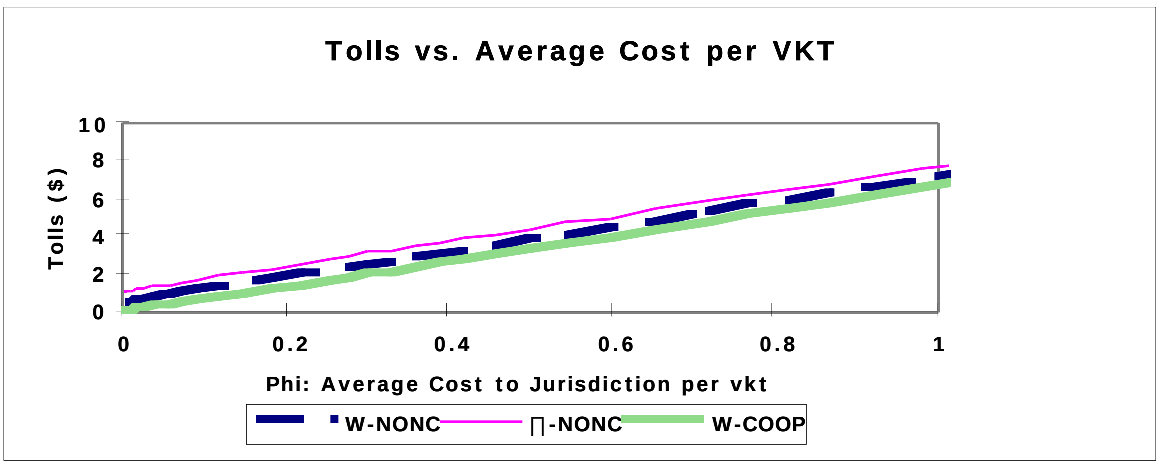

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Figure 4 Tolls as Average Trip Length Changes by Scenario

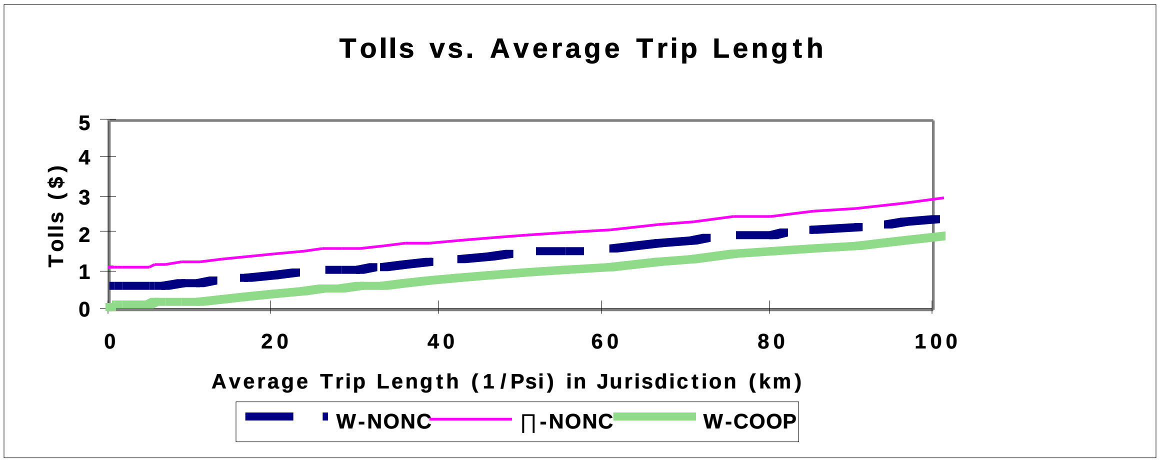
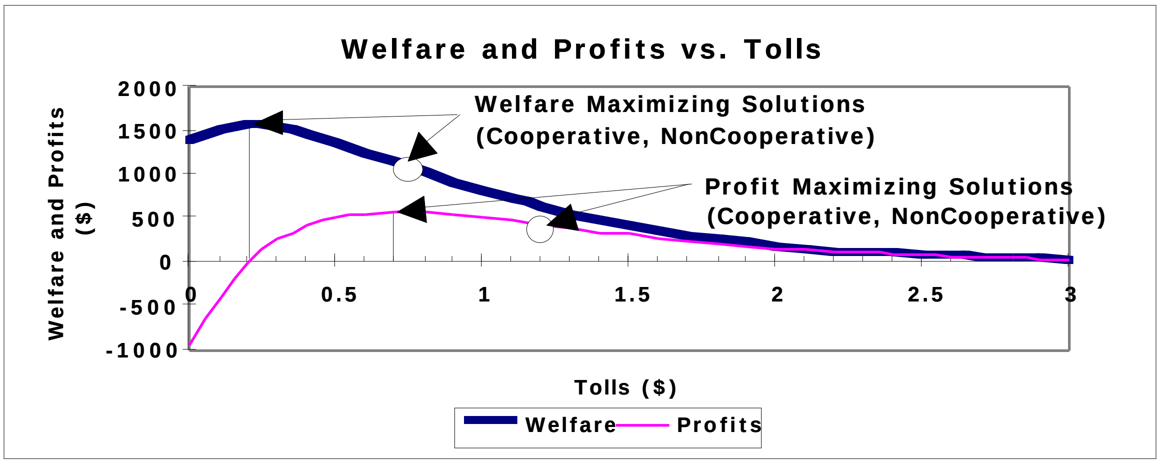


Figure 5 : Welfare and Profit as Tolls Change by Scenario



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[i] To quantify the importance of frontiers, of 133 major countries existing prior to the fall of the Soviet Union, there were 500 international boundaries between them, with each boundary containing multiple crossings (source: author's calculations). This does not include sub-national frontiers (state, provincial, county, or city boundaries, for instance).

[ii] By symmetry, the consumers' surplus in each direction is identical, and by symmetric trip tables, half the flow in each direction is made by residents, therefore we only need to compute the total consumers' surplus in one direction rather than half in both directions.

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[iii] The Nash equilibrium conditions state that when all jurisdictions are identical, each jurisdiction will try to achieve the highest welfare for themselves, recognizing that other jurisdictions will do the same. However in an indefinitely repeated prisoner's dilemma game, strategies which enforce cooperation by punishing "defection" can be employed to maximize overall welfare.

[iv] In an infinitely repeated games context, this is the best result that jurisdictions can attain over the long term, and though other solutions are also equilibria, no other solution improves on this one overall (though a single jurisdiction raising tolls - violating the equal tolls provision, may have a higher individual welfare or profit).

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